18.100C Homework 2

Due online by noon on Monday, September 28

This assignment has four parts. All four parts should be submitted in .tex and .pdf formats through the Stellar course website. Parts 1-3 should be in a single document and Part 4 in a separate document.

(1) For each of the following statements, write down the inverse, converse and contrapositive. Indicate which of the statements are logically equivalent to each other due to a simple rule of logic. State (but you need not prove) which statements are true and which are false.
   (a) If $r + s$ is rational then r is rational or s is irrational.
   (b) If r is irrational or s is irrational then $r + s$ is irrational.
   (c) If A is open then A^C is closed. (Take as given that A is in the universe of subsets of a given metric space X.)
   (d) If $0 < x < y$ then $0 < \frac{1}{y} < \frac{1}{x}$. (Take as given that x and y are elements of a given ordered field $\mathbb{F}$.)

(2) Write down the negation of each of the following statements.
   (a) $\forall \varepsilon > 0, x \in A, \exists y \in B$ such that $d(x, y) < \varepsilon$. (Take as given that A and B are subsets of some metric space (X, d).)
   (b) $\forall \varepsilon > 0\, \exists \delta > 0\, \forall f \in F, x \in X, y \in X \Big((d_1(x, y) < \delta) \implies (d_2(f(x), f(y)) < \varepsilon) \Big)$. (Take as given that F is a set of functions whose domain is the metric space (X, d_1) and whose range is another metric space (Y, d_2).)

(3) Construct the truth table for the statement $(P \wedge Q) \implies (P \vee Q)$.

(4) TeX up one of the solutions (your choice) to your regular 100B/C problem set from the assignment due Friday, September 25. (This will be part of every future recitation homework assignment.)